I have already shown (*ante*, xii, 406) that the radiating force at the Sun's surface, is directly comparable with the gravitating force at the surfaces of the Earth and Jupiter. This fact has suggested a reference of balancing forces to the distance (δ) from the Sun's centre at which the luminous and gravitating velocities would be equal, similar to my previous reference of the gravitating forces of different masses, to the distance from a planetary centre at which satellite and orbital velocities would be equal.

If we take Norton's values of the astronomical elements, and suppose the Sun's mass concentrated in a single point,

$$\delta = \frac{425{,}061.5}{(183{,}454 \div 265.52)^2} = .89041 \text{ miles.}$$

The circumference in which gravity would give a uniform velocity equivalent to that of light, is, therefore, 5.5946 miles, and each circular oscillation would be performed in $\frac{5.5946}{183{,}454} = \frac{1}{32791}$ second. The time of solar rotation, (2,174,425 seconds) is equivalent to (2,174,425 $\times$ 32791 =) 71,301,570,175 primary gravity-oscilliations (β).

The number of centres of luminous undulations in any sphere is proportioned to $\pi^3 r$; if we divide $\pi^3 \delta$ by β we obtain .0000245 in., which corresponds very closely with the wave length of the extreme red ray (.0000266).

Again, if we divide the time of solar rotation by $\pi \times$ the square of the time of a primary gravity-oscillation, we obtain

$$\left(2{,}174{,}425 \div \frac{\pi}{(32{,}791)^2} =\right) 744 \times 10^{12},$$

which corresponds nearly with the number of oscillations of the extreme violet ray in one second (727×10^{12}).

THE GAMUTS OF SOUND AND LIGHT.

By Pliny Earle Chase.

(*Read before the American Philosophical Society, March* 21, 1873.)

The evidences of planetary annuli of balancing forces may be still further supplemented by a comparison of the visible with the audible waves.

Various correspondences have been pointed out between the scale of color and musical scales, but Ponton has shown (Quarterly Journal of Science, No. XXXVII, pp. 91–103) that the agreement is not so close as has been sometimes supposed. There are, however, some interesting relations which seem to show an underlying harmonic law, which is partially obscured by want of homogeneity, either in the solar atmosphere, or in the æthereal medium, or in both.

Physical charts of the thermal equator give a mean temperature of about 82° F. Taking Ganot's (Dulong's) formula,

$$v = 1093\,(1 + .003665\,t)^{\frac{1}{2}},$$

and substituting the value $t = \frac{5}{9}\,(82° - 32°) = 27\frac{7}{9}°$, the mean equatorial velocity of sound should be 1147.3 ft. per second. The wave-length of the lowest *do* $\left(C_{-3}\right)$ is, therefore, $1147.3 \times 12 \div 16 = 860.5$ inches.

The following table gives the number of vibrations and the wave lengths of the fundamental note, for each of the first twenty-six octaves.

I. Table of Musical Octaves.

Note.	Vibrations per Second.	Wave Lengths.
C_{-3}	16	860.5
C_{-2}	32	430.25
C_{-1}	64	215.125
C	128	107.5625
C_2	256	53.78125
C_3	512	26.890625
C_4	1024	13.4453125
C_5	2048	6.72265625
C_6	4096	3.36132812
C_7	8192	1.68066406
C_8	16384	.84033203
C_9	32768	.42016601
C_{10}	65536	.21008300
C_{11}	131072	.10504150
C_{12}	262144	.05252075
C_{13}	524288	.02626037
C_{14}	1048576	.01313018
C_{15}	2097152	.00656509
C_{16}	4194304	.00328254
C_{17}	8388608	.00164127
C_{18}	16777216	.00082063
C_{19}	33554432	.00041031
C_{20}	67108864	.00020515
C_{21}	134217728	.00010257
C_{22}	268435456	.00005128
C_{23}	536870912	.00002564

The wave length of C_{23} corresponds very closely with that of the Fraunhofer C line (.00002586), and the corresponding octave is therefore directly comparable with the prismatic spectrum. Table II. contains a comparative exhibit of the wave-lengths, in hundred-millonths of an inch, o the twenty-sixth musical octave, with those of the Fraunhofer lines according to Angström's measurements, and the accompanying diagram graphically illustrates the comparison.

II. WAVE-LENGTHS OF LIGHT AND SOUND.

Fr. Lines.	Length.	Musical Notes.	Length.
A	2997	A	3077
B	2707	B	2735
C	2586	C	2564
D′′	2322	D	2279
E	2076	E	2051
F	1915	F	1923
G	1697	G	1709
H_2	1550	A	1538
Mean, Arith.	2231	Mean, Arith.	2234
" Geo.	2180	" Geo.	2180

By a singular fortuitous coincidence, the same names have been given to the principal nodes in the luminous octave, and to the corresponding notes of the Diatonic Scale. Although the differences between the analogous wave-lengths are too great to be strictly harmonized with any known musical scale there can be no hesitation in any single instance, as to the proper notes for comparison. The tendency to minor intervals in the spectrum, the closeness of the agreement at F, which has been called "the key-note of nature," and the equality of the means, may, possibly, have something more than a merely curious significance. Analogies of this description, based on the lines which are most prominent, are much more satisfactory than those between the more minute and more numerous microscopic lines.

All musical scales being more or less artificial, one might, perhaps, be constructed, which should agree more closely with the luminous gamut than any hitherto proposed. Table III. exhibits some interesting features of reciprocal resemblances to Poole's "Double Diatonic Scale." The left hand column gives the number of vibrations of the successive musical notes; the middle column, the wave-lengths of the Fraunhofer lines; the right hand column, a set of wave-lengths in which the intervals are the same as Poole's but differently arranged and not fitted for harmony.

III. LUMINOUS AND DOUBLE DIATONIC SCALES.

Poole's Notes.	Fr. Lines.	False Scale.
3096	2997	3000
2709	2707	2700
2580	2586	2571
2322	2322	2314
2064	2076	2057
1935	1915	1928
1741	1697	1714
1548	1550	1543

In six out of eight notes, the agreement with Poole's Scale is remarkably close. The differences from the False Scale are all slight, in no case exceeding one per cent., but this merely shows that the *successive* lines, are arranged nearly at such intervals as *may be* used in harmony. Table IV. shows more satisfactory agreements between remoter notes, in Table II.

IV. ACCORDANT INTERVALS IN LIGHT AND SOUND.

	Light.	Sound.	
BF	1.413	1.422	BG ÷ FG = CE and 2 Semitones.
BG	1.595	1.600	$\frac{8}{5}$ = Minor Sixth.*
CE	1.245	1.250	$\frac{5}{4}$ = Major Third.
CA	1.669	1.667	$\frac{5}{3}$ = Sixth.†
DE	1.118	1.111	$\frac{10}{9}$ = Minor Tone.
EA	1.341	1.333	CA ÷ CE = Fourth.
FG	1.129	1.125	$\frac{9}{8}$ = Major Tone.

Ponton's explanation of such harmonies as these, "that the ratios are those of the respective amounts of *vis inertiæ* possessed by the vibrating atoms which originate the lines,"‡ seems quite satisfactory; but I can hardly agree with him in believing that "their arithmetical coincidence with certain musical intervals is merely accidental, and such as might be expected, according to the law of probabilities, when so large a number of lines are concerned." His conclusion, however, was based upon observations of the more minute lines, to which I have already referred, as less satisfactory. I know of no law of probability which would explain such close approximations as are shown in Tables II., III. and IV., without supposing some kind of harmonical dependence upon one fundamental vibration.

The existence of analogous harmonies in atomicities, chemical composition, phyllotaxis, and planetary relations, suggests the hypothesis that the *vis inertiæ* itself may be determined by harmonic vibrations. The two forces which are commonly exclusively considered in explaining orbital revolution, are centripetal gravitation, and tangential inertia. There must, also, be a force of centrifugal emanation which enables the Sun to radiate his continual supply of light and heat, and I believe that a proper application of mathematical analysis in the investigation of such a force, would open a wide field for interesting and valuable research.

If the ultimate particles of bodies are mathematical points, the potential energies of cosmical globes are proportional to the products of their masses by their radii. The density of particles in elastic fluids varying as the squares of the times of molecular diffusion, let us suppose that the time of diffusion is determined by the ratio of orbital time, to time of fall to an attracting centre ($\sqrt{32}$). Then, if l represent the length of elastic undulation, we may have some reason to look for the following proportionality :

$$mr \propto \frac{1}{l} \propto vd \propto vt^2 \propto 32\,v.$$

* Or, Fourth and Minor Third. † Or, Fourth and Major Third.

‡ Loc. cit. p. 100.

Let us assume the correctness of the following values :

Constant of aberration (Struve).....................	497.827
Theoretical velocity of sound (Enc. Metrop.)........	916.322
Mass, Earth ÷ Sun (Newton).........................	$4.432 \left(\frac{\text{parallax}}{1000}\right)^3$
Earth's radius (Norton)...........................	3962.8

The foregoing proportionality will then yield the following results :

Velocity (Light ÷ Sound) $= \frac{1}{32} C_{23} \div C_{-3} = 32^4 =$ $mr \div 32 =$	1,048,576
Velocity of Light (1,048,576 × 916.322 ÷ 5280)......	181,976
Distance of Sun (181,976 × 497.827).................	90,592,565
Parallax (1,296,000″ × 3962.8) ÷ (2π × 90,592,565)..	9″.0227
Mass, 4.432 (par ÷ 1000)^3...........................	1 ÷ 307,178
Mass × radius, (Sun ÷ Earth) = 32^5.................	33,554,432
Radius (Sun ÷ Earth), 33,554,432 ÷ 307,178.........	109.23
Sun's semi-diameter, 9″.0227 × 109.23..............	985″.5

The most careful estimates of the Sun's actual semi-diameter vary between 961″.82 (Br. Naut. Almanac), and 962″.1025 (Lockyer), being about $2\frac{1}{2}$ per cent. less than the above theoretical value. In consideration of the fact that no allowance has been made for orbital eccentricity, æthereal condensation, mean atmospheric elasticity, or other modifying influence, so close an approximation is very satisfactory. If the theoretical velocity of sound were increased $\frac{5}{8}$ of one per cent. in consequence of such allowances, we should have : Sun's distance, 91,158,769 ; parallax, 8″.966 ; mass, 312,973 ; radius, 107.21 ; semi-diameter, 961″.25. The most recent astronomical estimates of solar distance range between 91,186,000 (Winnecke), and 92,380,000 (Newcomb).

In a former communication (*ante*, xii. 399), I assigned 91,965,500 miles as a theoretical upper limit of the Earth's possible mean distance from the Sun. For reasons which seem to me still more conclusive, I am inclined to regard the above approximation of 90,592,565 miles as the lowest possible limit, and I shall look with much interest and confident expectation, for a confirmation of these estimates by the observations of the approaching transit of Venus.

The inverse proportionality of velocity to wave-length suggests a further extension of the comparison. Airy's estimate of the time required for the solar disturbance of July 7th, 1872, to affect the terrestrial magnetism, would make the luminous velocity (2 *h.* 20 *m.* ÷ 497.827 *s.* =) 16.87 × the magnetic. This estimate would be increased if the outburst took place before it was observed by Father Secchi, but it is not to be supposed that it could be doubled. If the Sun's distance is 90,592,565 miles, the theoretical explosive range of hydrogen should be 4 (90,592,565 ÷ 307,198) = 1179.7 ; (3962.8 + 1179.7) ÷ 294.92 = 17.4. This appears to indicate a magnetic gamut, four octaves below that of light, bearing the same relations to the luminous octave, in frequency of undulation, as a single vibration of a pendulum bears to the lowest number of vibrations which will produce a continuous musical note. If this ratio should be confirmed by further observations, it will furnish a valuable addition to the numerical magnetic relations which I have already pointed out (*ante*, x., 358, &c.).

A tendency to four-octave intervals, is shown in the different planetary values of mr (Sun ÷ planet) which are, approximately as follows:

Mercury............	$2^{30} \times x$*	Jupiter..............	$2^{13} \times \frac{5}{4}$ (E)
Venus...............	$2^{25} \times \frac{4}{3}$ (F)	Saturn..............	$2^{15} \times \frac{5}{4}$ (E)
Earth...............	2^{25} (C)	Uranus..............	$2^{19} \times \frac{6}{5}$ (D ♯)
Mars.................	2^{29} (C)	Neptune............	$2^{18} \times \frac{5}{3}$ (A)

The value for Mercury is 3×4 octaves higher than that for Neptune; Venus and Earth, 3×4 octaves higher than that of Jupiter; Mars, 4 octaves higher than Earth; Uranus, 4 octaves higher than Saturn.

Stated Meeting, March 7th, 1873.

Present 22 members.

Vice-President, Mr. Fraley, in the Chair.

A letter accepting membership was received from Geo. Burrows, M. D., F. R. S., Prest. R. College of Physicians, London, 18 Cavendish Square, February 12, 1873.

Letters acknowledging receipt of publications were read from the Society of Sciences at Geneva (XIV iii, 86, 87), Nov. 1, 1872; from the Society at Wiesbaden (78 to 87), Sept. 10, 1872; from the R. Bavarian Academy (86 to 87 bis; XIV. iii, bis), Dec. 1, 1872; Prof. R. Owen (84, 85), Dec. 1, 1871; and the Smithsonian Institute (89), Feb. 15, 1873.

Letters of envoy were received from the R. Bavarian Academy, asking also for Transactions (XII ii, iii.); from the R. Society London, and Linæan Society asking for Transactions (III, pp. 315 to 382), and Proceedings (62 and 87,); And from the S. P. S. Geneva.

On motion, the Librarian was authorized to forward the required publications if possible, and to request in return a copy of the Catalogue of Memoirs published by the Royal Society.

* The mass of Mercury is so uncertain, that the note corresponding to the C line cannot be determined.